TECHNICAL NOTE:
THE MIDPOINT DEFINITION OF ELASTICITY

1 Definition

In the slides, we have defined the price elasticity of demand (when moving from one point to another along the demand curve) as the ratio between the percentage change in the quantity demanded and the percentage change in price. Moreover, we have translated this definition into a mathematical formula by using the **midpoint definition of percentage change**. That is, we have defined the percentage change in a generic variable X as the ratio between the discrete change of X ($\Delta X = X_2 - X_1$) and the average of the old and new values of X ($\bar{X} = [X_1 + X_2]/2$). The resulting formula is:

$$\epsilon_d = \frac{(Q_2 - Q_1)}{(Q_1 + Q_2)/2} / \frac{P_2 - P_1}{(P_1 + P_2)/2} = \frac{(Q_2 - Q_1)(P_1 + P_2)}{(P_2 - P_1)(Q_1 + Q_2)}.$$

2 Motivation

The above definition of percentage change is different from traditional (and more intuitive) methods of calculating percentage changes. For instance, under traditional methods, a price that rises from 10 to 12 is a 20 percent increase. This is calculated as the ratio between the price change of 2 ($= 12 - 10$) and the initial value of 10. That is: $2/10 = 0.20$ (20 percent). Using the midpoint approach, the discrete change is still 2, but instead of using the initial value of 10, the base value is the average of 10 and 12, that is 11 ($= [10 + 12]/2$). This approach results in a change of about 18 percent: $2/11 = 0.18$.

Although the midpoint method seems like a way of making a simple task excessively complicated, this is not the case. This method is actually intended to generate **identical elasticity calculations** over a given segment of a curve, whether the variables increase or decrease. The below example will illustrate this point.

3 Example

If you try to calculate the price elasticity of demand between two points on a demand curve using the traditional definition of percentage change, you will quickly notice the annoying problem discussed above: The elasticity from point A to point B seems different from the elasticity from point B to point A. Consider these numbers:

	Price	Quantity
Point A:	4	120
Point B:	6	80

Using the traditional definition, going from point A to point B, the price rises by 50 percent ($2/4 = 0.50$), and the quantity falls by 33 percent ($40/120 = 0.33$), indicating that the price elasticity of demand is 33/50, or 0.66. By contrast, going from point B to point A, the price falls by 33 percent ($2/6 = 0.33$), and the quantity rises by 50 percent ($40/80 = 0.50$), indicating that the price elasticity of demand is 50/33, or 1.5.

One way to avoid this problem is to use the midpoint method for calculating elasticities. Rather than computing the percentage change using the traditional way (i.e., dividing the change by the initial level), the midpoint method computes a percentage change by dividing the discrete change by the midpoint of the initial and final levels. For instance, 5 is the midpoint of 4 and 6. Therefore, according to the midpoint method, a change from 4 to 6 is considered a 40 percent rise ($2/5 = 0.40$). Similarly, a change from 6 to 4 is considered a 40 percent fall. Because the midpoint method gives the same answer regardless of the direction of change, it is often used when calculating the price elasticity of demand between two points. According to the midpoint approach, when going from point A to point B, the price rises by 40 percent ($2/5 = 0.40$), and the quantity falls by 40 percent ($40/100 = 0.40$). Similarly, when going from point B to point A, the price falls by 40 percent, and the quantity rises by 40 percent. In both directions, the price elasticity of demand equals 1.