TECHNICAL NOTE: PLAYING AROUND WITH DEMAND AND SUPPLY LINES

1 The problem

At first, those of you who know a little of coordinate geometry may find themselves at odds with the supply-and-demand diagram. You have heard in class that "every point on the demand curve represents the overall quantity of the good demanded by consumers *at every given price*" and that, similarly, "every point on the supply curve represents the overall quantity of the good supplied by firms *at every given price*". In both cases, the quantity is thought of as the **dependent variable** and the price as the **independent variable**. That is, quantity depends on price. Hence, you would expect to see the quantity on the vertical axis and the price on the horizontal axis. But in economics it is exactly the other way around: In the basic supply-and-demand diagram, price is on the vertical axis and quantity on the horizontal axis. Why are they switched? It turns out that the motivation is more historical than logical: The economists who formalized this model drew the diagram in such a way.

The partial equilibrium supply-and-demand model was originally developed by **Antoine Cournot** (in 1838) and then broadly publicized by **Alfred Marshall** (in 1890). But why did they put price on the vertical axis? The famous Harvard economist **Gregory Mankiw** advances this explanation:

> "The early economists may have been imagining that, in the very short run, a given quantity of goods was supplied to the market (an agricultural harvest, for example). The supply curve is then vertical, and the price adjusts to ensure that quantity demanded equals this exogenous quantity supplied. So, in this very short run, the price seems more like the dependent variable. Now, however, the choice of axes is based more on historical convention than logic."

Another famous Harvard economist, **Robert Barro**, suggests a different explanation:

> "As I recall, Hicks in Value and Capital thought in terms of demand price and supply price. The demand price is how much a person was willing to pay for an additional unit of goods (starting from some initial quantity, Q). The supply price is how much a producer would have to be paid to provide an additional unit of goods. This construction–which I think comes from Marshall–makes it natural to have P on the vertical axis and Q on the horizontal."

Whatever the reason, the convention to put the price (P) on the vertical axis and the quantity (Q) on the horizontal axis is now the common standard in economics. And, at first, this can produce mistakes. Consider the generic equation of the demand line:

$$Q = a - bP \qquad (a, b > 0). \tag{1}$$

When we say that a shift to the right of the demand line can be represented as an increase of the vertical intercept, we do not have in mind the intercept a, but the intercept in the graph with P on the vertical axis, i.e., (a/b). The two examples below will illustrate this point and (hopefully) help to avoid confusion.

2 A first example: The demand shift caused by a subsidy

Consider the demand line in equation (1). This can be rewritten as:

$$P = \frac{a}{b} - \frac{1}{b}Q \tag{2}$$

which is the line that we graph in the usual diagram (with P, the variable on the vertical axis, expressed as a function of Q). See **Figure 1** at the end of this note. Now, consider the effect of a **subsidy** paid to consumers to buy this good. As we know, this is equivalent to a shift to the right (or upward) of the demand line, exactly equal to the amount of the subsidy. That is:

$$P = (\frac{a}{b} + s) - \frac{1}{b}Q \tag{3}$$

In the usual economics version, equation (3) can be rewritten as:

$$Q = (a + sb) - bP \tag{4}$$

i.e., the standard demand line - equation (1) - with an increased (horizontal) intercept $(a + sb)$ instead of a.

From an intuitive point of view, the same result (i.e., a formal expression for the new after-subsidy demand line) can be derived from the following reasoning. After the subsidy, consumers still respond to the previous market conditions (i.e., they respond to the paid price according to parameters a and b), but now the price they effectively pay is not the final price in the market: they pay the market price minus the subsidy (offered by the government). In other terms, the new demand line is:

$$Q = a - b(P - s) = (a + sb) - bP \tag{5}$$

which is exactly equation (4). See again **Figure 1** for a better understanding of this example.

3 A second example: The supply shift caused by a tax

Consider the following generic expression for a supply line:

$$Q = a + bP \qquad (b > 0). \tag{6}$$

Now, the parameter a can be either positive or negative. This equation can be rewritten as:

$$P = -\frac{a}{b} + \frac{1}{b}Q \tag{7}$$

which is the line that we graph in the usual diagram (with P, the variable on the vertical axis, expressed as a function of Q). See **Figure 2** at the end of this note for an example with $a < 0$. Now, consider the effect of a **sales tax** paid by the sellers of this good. As we know, this is equivalent to a shift to the left (or upward) of the supply line, exactly equal to the amount of the tax. That is:

$$P = (-\frac{a}{b} + t) + \frac{1}{b}Q \tag{8}$$

In the usual economics version, equation (8) can be rewritten as:

$$Q = (a - tb) + bP \tag{9}$$

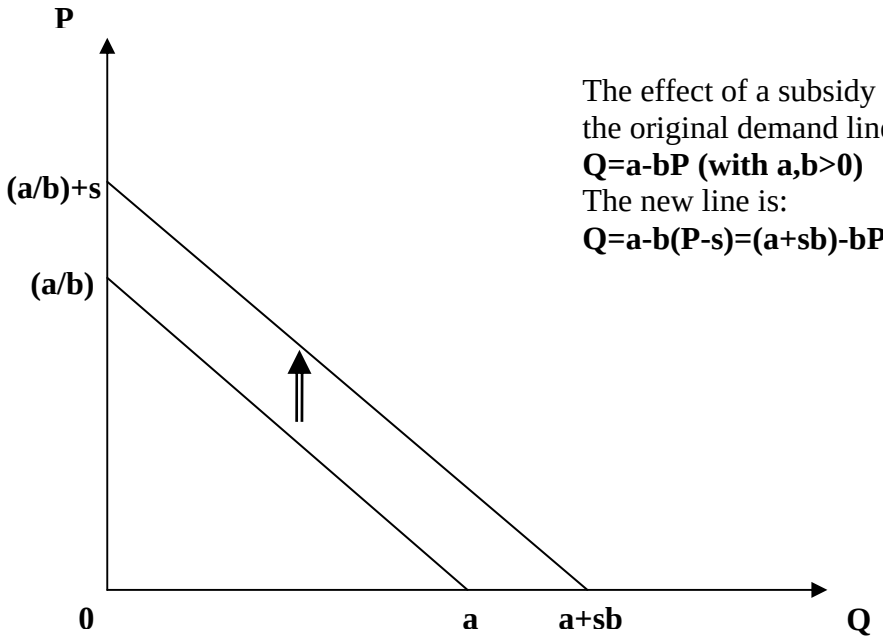
i.e., the standard supply line - equation (6) - with a decreased (horizontal) intercept $(a - tb)$ instead of a.

From an intuitive point of view, the same result (i.e., a formal expression for the new after-tax supply line) can be derived from the following reasoning. After the tax, sellers still respond to the previous market conditions (i.e., they respond to the collected price according to parameters a and b), but now the price they effectively cash in is not the final price in the market: they get the market price minus the tax (collected by the government). In other terms, the new supply line is:

$$Q = a + b(P - t) = (a - tb) + bP \tag{10}$$

which is exactly equation (9). See again **Figure 2** for a better understanding of this example.

FIGURE 1



The effect of a subsidy on the original demand line:
Q=a-bP (with a,b>0)
The new line is:
Q=a-b(P-s)=(a+sb)-bP

FIGURE 2

The effect of a tax on the original supply line:
Q=a+bP (with a<0, b>0)
The new line is:
Q=a+b(P-t)=(a-tb)+bP