Government 2005
Formal Political Theory I
Lecture 4

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Lecture 4: Overview

- Electoral competition with office-seeking candidates (recap)
- Electoral competition with policy-oriented candidates
- Policy-oriented candidates and uncertainty
- Office-seeking candidates and valence advantage
- Addendum (if we have time):
  - Recap and extensions on market power

Office-seeking candidates

- 2 political parties or candidates compete for one elected office, by choosing positions on policy (x)
- There is no commitment problem
- We assume parties care only about winning
- There are *n* non-strategic voters, with *n* odd
- Each voter has single-peaked preferences, and is assumed to vote for the party whose position the voter prefers
- The party that receives the most votes wins the election
- If indifferent, the voter abstains
- If there is a tie, a fair coin is tossed to determine the winner

Office-seeking candidates (contd.)

- First, Let v_1 be the number of votes received by party 1; then $v_2 = n - v_1$
- The payoff to party 1 is given by:

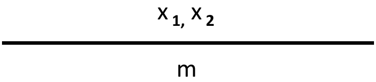
$$\pi_1(x_1, x_2) = \begin{cases} 0 & \text{if } v_1 < v_2 \\ W/2 & \text{if } v_1 = v_2 \\ W & \text{if } v_1 > v_2 \end{cases}$$

and $\pi_2(x_1, x_2) = 1 - \pi_1(x_1, x_2)$, with $W \geq 0$

- There is a unique Nash equilibrium to the game, in which both parties locate at the median of the voters' ideal points, m
- This is a variant of the median voter theorem

Office-seeking candidates (contd.)

- We first prove that $(x_1, x_2) = (m, m)$ is a Nash equilibrium
- Let z be a "typical" voter's ideal point
- Let m be the median of the voters' ideal points



- If $(x_1, x_2) = (m, m)$, then $v_1 = v_2$, and $\pi_1 = \pi_2 = 1/2$
- No incentive to deviate: $W/2 \to 0$. Therefore (m, m) is Nash

Office-seeking candidates (contd.)

- To prove uniqueness we must consider *all* cases $(x_1, x_2) \neq (m, m)$
- If $x_1 \neq x_2$, we may have two sub-cases:
  - i loses and therefore she will have incentive to deviate to m: either $0 \to W/2$ (if $x_j = m$) or $0 \to W$ (if $x_j \neq m$)
  - there's a tie and therefore both candidates have incentive to deviate to m: $W/2 \to W$
- If $x_1 = x_2 \neq m$:
  - there's a tie and therefore both candidates have incentive to deviate to m: $W/2 \to W$

Policy-oriented candidates

- Suppose parties care both about policy and about winning
- Assume that the policy preferences of all voters and parties can be described by ideal points in a quadratic loss function
- In particular, suppose party 1's payoff is given by:

$$\phi_1(x_1, x_2) = \begin{cases} -(z_1 - x_2)^2 & \text{if } v_1 < v_2 \\ W/2 - (z_1 - \frac{1}{2}(x_1 + x_2))^2 & \text{if } v_1 = v_2 \\ W - (z_1 - x_1)^2 & \text{if } v_1 > v_2 \end{cases}$$

- When there is a tie the two parties "share office"
- Party 2's payoff function is the obvious analog
- Also, suppose the parties' ideal points are on opposite side of the median voter, that is $z_1 < m < z_2$ (how innocent?)
- To make argument extreme, assume $W = 0$ in the following

Policy-oriented candidates (contd.)

- There is a unique Nash equilibrium, with $(x_1, x_2) = (m, m)$
- The proof proceeds as above
- Intuition: Although each candidate would like to choose policies close to her ideal point, she cannot because if she chooses a policy other than the median of the voters' ideal points then she loses the election
- Proof of existence: no incentive to deviate for i because payoff would be $-(z_i - m)^2$ anyway; and deviation would not be equilibrium as m is not best response by j to $x_i \neq m$
- Formally: $m \in b_i(m)$ but $m \notin b_j(x_i \neq m)$
- Important to note that this holds even if the candidates care *only* about policy, that is, even if $W = 0$ (as we are assuming right now)

Policy-oriented candidates (contd.)

- To prove uniqueness we must consider *all* cases $(x_1, x_2) \neq (m, m)$
- (Case I) $x_2 > x_1$
  - If $x_1 < x_2 < m$: candidate 2 has incentive to deviate toward m, $-(z_2 - m)^2 > -(z_2 - x_2)^2$
  - If $m < x_1 < x_2$: candidate 1 has incentive to deviate toward m, $-(z_1 - m)^2 > -(z_1 - x_1)^2$
  - If $x_1 < m < x_2$:
    - If $v_1 > v_2$: candidate 2 has incentive to deviate toward m, $-(z_2 - m)^2 > -(z_2 - x_1)^2$
    - If $v_1 < v_2$: candidate 1 has incentive to deviate toward m, $-(z_1 - m)^2 > -(z_1 - x_2)^2$
    - If $v_1 = v_2$: both candidates have incentive to deviate toward m by ϵ small enough

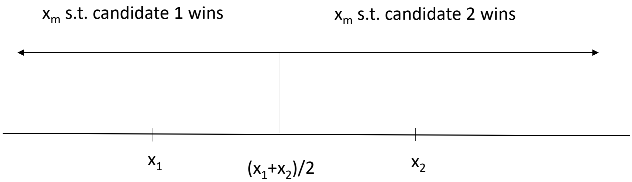
Policy-oriented candidates (contd.)

- (Case II) $x_2 = x_1 \neq m$
  - If $x_2 = x_1 > m$: candidate 1 has incentive to deviate toward m
  - If $x_2 = x_1 < m$: candidate 2 has incentive to deviate toward m
- (Case III) $x_2 < x_1$
  - Straightforward to rule out as winner would prefer to loose
- And this makes the trick as we have considered all profiles

Policy-oriented plus uncertainty

- Let's consider an extreme/simplified version of policy bias (leaving loss functions as exercise): candidate 1 (2) wants x to be as far as to the left (right) as possible
  - This is an extreme assumption, but it's the famous "law of plus one:" no matter how intense is your effort to please extremist politicians, they are always going to ask for more, i.e., "plus one" (don't google this up, I'm just kidding... although this is sadly true with most extremist politicians)
- But there's uncertainty about the location of the median voter's bliss point
- Ex ante: x_m random variable with cdf $F(x_m)$ and pdf $f(x_m)$
- $Prob\{1 \text{ wins}\} = F[(x_1 + x_2)/2]$
- $Prob\{2 \text{ wins}\} = 1 - F[(x_1 + x_2)/2]$

Policy-oriented plus uncertainty (contd.)



- Expected policy:

$$E[x] = x_1 F\left(\frac{x_1 + x_2}{2}\right) + x_2\left[1 - F\left(\frac{x_1 + x_2}{2}\right)\right]$$

- Again $x_1 \leq x_2$. Best responses comes from:

$$\underset{x_1}{Min}\left\{x_1 F\left(\frac{x_1 + x_2^*}{2}\right) + x_2^*\left[1 - F\left(\frac{x_1 + x_2^*}{2}\right)\right]\right\}$$

$$\underset{x_2}{Max}\left\{x_1^* F\left(\frac{x_1^* + x_2}{2}\right) + x_2\left[1 - F\left(\frac{x_1^* + x_2}{2}\right)\right]\right\}$$

Policy-oriented plus uncertainty (contd.)

- From FOCs:

$$\frac{x_2^* - x_1^*}{2} f\left(\frac{x_1^* + x_2^*}{2}\right) = F\left(\frac{x_1^* + x_2^*}{2}\right)$$

$$\frac{x_2^* - x_1^*}{2} f\left(\frac{x_1^* + x_2^*}{2}\right) = 1 - F\left(\frac{x_1^* + x_2^*}{2}\right)$$

- Therefore: $F\left(\frac{x_1^*+x_2^*}{2}\right) = \frac{1}{2}$

$$x_2^* - x_1 * = \frac{1}{f\left(\frac{x_1^*+x_2^*}{2}\right)}$$

- Possible interpretation: Centrist voters' force of attraction

Policy-oriented plus uncertainty (comparative statics)

- Assume: $x_m \sim N(m, \sigma^2)$
- $F(x) = 1/2$ iff $x = m$

$$x_2^* - x_1^* = \frac{1}{f(m)} = \sqrt{2\pi\sigma^2}$$

$$x_2^* = m + \sqrt{(\pi\sigma^2)/2} \qquad x_1^* = m - \sqrt{(\pi\sigma^2)/2}$$

- Bottom line: the larger σ^2, the larger $(x_2^* - x_1^*)$, i.e., more divergent policy platforms
- Exercise (1): what if $x \in [0, 1]$ and $x_m \sim U[0, 1]$?
- Exercise (2): what if parties' policy preferences are expressed by loss functions as before? That is: $-(z_i - x)^2$

Office-seeking plus valence advantage

- Suppose again that parties care only about winning office
- Seemingly minor changes in the payoff structure can have large effects on the equilibrium behavior
- Suppose that party 2 has a "valence" advantage over the other: each voter with ideal point at z receives utility $-(z-x_1)^2$ if party 1 wins, and utility $-(z-x_2)^2+k$ if party 2 wins, where $k>0$ is the valence component
- Also suppose that party 2 wins office in case of a tie in votes
- Party 1's payoff is

$$\pi_1(x_1,x_2)=\begin{cases}0 & \text{if } v_1 \leq v_2 \\ 1 & \text{if } v_1 > v_2\end{cases}$$

and $\pi_2(x_1,x_2)=1-\pi_1(x_1,x_2)$

Office-seeking plus valence advantage (contd.)

- Then there are *many* Nash equilibria to this game
- Suppose party 1 chooses $x_1 = m$ (the best it can do!)
- Then $v_2 > v_1$ if and only if

$$\begin{aligned} & -(m - x_2)^2 + k \geq -(m - x_1)^2 = 0 \\ \text{or} \quad & |m - x_2| \leq \sqrt{k} \\ \text{or} \quad & m - \sqrt{k} \leq x_2 \leq m + \sqrt{k} \end{aligned}$$

- If party 2 locates in the interval $[m - \sqrt{k}, m + \sqrt{k}]$, it wins the election with probability one
- If it locates outside this interval then party 1 can win by choosing $x_1 = m$

Office-seeking plus valence advantage (contd.)

- Any pair (x_1^*, x_2^*) such that $x_2^* \in [m - \sqrt{k}, m + \sqrt{k}]$ and any x_1 constitutes a Nash equilibrium
- Party 2 always wins the election in equilibrium
- Are there other equilibria? Indeed, the threat of $x_1 = m$ constraints the set of equilibrium x_2
- Exercise (3): what if parties have bliss points (z_1, z_2) in addition to the valence advantage of party 2?

Where are we?

- We have studied **static games of complete information**
- References:
  - Lecture slides $\rightarrow$ 1 through 4 (final folder)
  - Osborne $\rightarrow$ chapters 1 through 4 (excluding 3.5, 3.6, 4.6) + 12
  - McCarty & Meirowitz $\rightarrow$ chapter 5
  - Gibbons $\rightarrow$ chapter 1
- For next class, please come with:
  - Doubts (on what we've discussed)
  - Curiosities (on what we've briefly mentioned)
- Then, we'll move on to **dynamic games of complete information**

Addendum: Game-theoretic analysis of market power

- Cournot duopoly (recap)
- Cournot oligopoly with n firms
- Bertrand duopoly with homogeneous product
- Bertrand duopoly with differentiated products

Cournot duopoly

- You have solved/discussed this with Jeremy in sections
- If $P = a - Q = a - q_1 - q_2$ *and* $c_i(q_i) = cq_i$ (no fixed costs and constant marginal costs) *and* firms compete on quantity
- Then: $q_1^* = q_2^* = (a - c)/3$ is the unique Nash equilibrium
- Firms would like to collude on monopolistic quantity $(a - c)/2$ but unsustainable as equilibrium

Cournot oligopoly with n firms

- Firms compete on quantity
- $P = a - Q$ with $Q = q_1 + ... + q_n$
- Again $c_i(q_i) = cq_i$
- Profits: $\pi_i = (P - c)q_i = (a - Q - c)q_i$
- Best responses can be obtained by FOCs of:

$$\overset{Max}{q_i} \; \{ (a - \sum_{j \neq i} q_j^* - q_i - c) q_i \}$$

$$a - c = \sum_{j \neq i} q_j^* + 2q_i^*$$

- All firms are symmetric $\Rightarrow q_j^* = q_i^* = q^*$

$$a - c = (n+1)q^* \qquad q^* = \frac{a-c}{n+1}$$

$$Q^* = \frac{n(a-c)}{n+1} \qquad P^* = a - \frac{n(a-c)}{n+1}$$

- Therefore: as $n \to \infty$, $P^* \to c$

Bertrand duopoly with homogeneous product

- Firms i and j compete on price: $p_i, p_j \geq 0$
- Again $c_i(q_i) = cq_i$
- Quantities sold in equilibrium are function of price competition:
  - $q_i = (a - p_i)$ if $p_i < p_j$
  - $q_i = (a - p_i)/2$ if $p_i = p_j$
  - $q_i = 0$ if $p_i > p_j$
- Because of discontinuity in payoffs we cannot solve with best responses from maximization; we must look at all profiles

Bertrand duopoly with homogeneous product (contd.)

- (I) $p_i^* > p_j^* = c$ is not Nash
  - j can increase π_j by deviating to $p_j^* + \epsilon$ with $\epsilon < p_i^* - p_j^*$
- (II) $p_i^* > p_j^* > c$ is not Nash
  - i can make positive profits by deviating to $p_j^* - \epsilon > c$
- (III) $p_i^* = p_j^* > c$ is not Nash
  - i can increase profits by deviating to $p_i^* - \epsilon > c$ for small ϵ
- (IV) $p_i^* = p_j^* = c$ is (unique) Nash equilibrium
  - $\pi_i = \pi_j = 0$ but no incentive to deviate
  - Lower price means negative profits
  - Higher price means zero profits anyway
  - Stark result: Competitive outcome with just two firms

Bertrand duopoly with differentiated products

- Firms i and j sell different product but there's some degree of substitutability between the two
- $q_i(p_i, p_j) = a - p_i + bp_j$
  - where $b > 0$ measures substitutability
- Again $c_i(q_i) = cq_i$
- Again i and j simultaneously set their prices: $p_i, p_j \geq 0$
- Best responses can be obtained by FOCs of:

$$\underset{p_i \geq 0}{Max} \{ (a - p_i + bp_j^*)(p_i - c) \}$$

Bertrand duopoly with differentiated products (contd.)

- From FOCs:

$$a - p_i^* + bp_j^* - p_i^* + c = 0$$

$$p_1^* = \frac{a + bp_2^* + c}{2} \qquad p_2^* = \frac{a + bp_1^* + c}{2}$$

- Solving this system of two equations:

$$p_1^* = p_2^* = \frac{a + c}{2 - b}$$

- Because of less than perfect substitutability, firms enjoy some market power and we no longer converge to the competitive benchmark where prices are equal to marginal costs